Algebra III

Lesson 30

Addition of Vectors – Overlapping Triangles
Vector Review

Vectors are in two parts – Size and Direction

Writing Vectors

Polar Form - \( r/\theta \) or \((r,\theta)\)

Rectangular Form - \( x\hat{i} + y\hat{j} \) or \((x,y)\)

Note: Vectors cannot be added in polar form unless the angles are 180° apart.

Otherwise, vectors must be in or converted to rectangular form
Samples:

$7/33^\circ + 4/100^\circ$

1) Draw 1$^{\text{st}}$ vector

2) Draw 2$^{\text{nd}}$ vector as a continuation from the end of the 1$^{\text{st}}$ vector.

$12/90^\circ + 15/270^\circ$

3) The answer goes from the beginning of the 1$^{\text{st}}$ vector to the end of the 2$^{\text{nd}}$ vector.
\((7\hat{i} + 6\hat{j}) + (-4\hat{i} + 2\hat{j})\)
Steps to Adding Vectors

1) If both vectors are already in rectangular form, go to step 5.

2) If both vectors are in polar form, check to see if the angles are 180° apart. If they are not go to step 4.

3) Subtract the smaller length from the larger length to determine the final length. Use the angle of the larger vector for the final direction.

Example: (12,90°) (15,270°)

15-12=3 (3,270°)
Steps to Adding Vectors

4) Convert the vectors to rectangular form. (This we have done many times. Only need the primary version, not all four.)

5) Combine the x pieces with each other, and combine the y pieces. (See Example C.)

6) If necessary, put this combined vector back into polar form. (This is also something we have done.)
Example 30.1

Find the resultant (answer) of $4/20° + 7/-230°$.

1) Not in rectangular form.

2) $-230 \rightarrow 360 - 230 = 130$  
   $130 - 20 = 110$  
   Not 180 apart.

3) Skip

4) $\frac{4}{20°}$ and $\frac{7}{-230°}$

\[ \begin{align*}
\sin 20° &= \frac{y}{4} \\
y &= 4 \sin 20° \\
y &= 1.37 \\
\cos 20° &= \frac{x}{4} \\
x &= 4 \cos 20° \\
x &= 3.76
\end{align*} \]

(3.76, 1.37)

\[ \begin{align*}
\sin 50° &= \frac{y}{7} \\
y &= 7 \sin 50° \\
y &= 5.36 \\
\cos 50° &= \frac{x}{7} \\
x &= 7 \cos 50° \\
x &= 4.50
\end{align*} \]

(-4.50, 5.36)
5) \((3.76)+(-4.50)=-.74\)

\((1.37)+(5.36)=6.73\)

\((- .74, 6.73)\) or \(- .74\hat{i} + 6.73\hat{j}\) Possible final answer, but not this time.

6) \[ r^2 = .74^2 + 6.37^2 \]

\[ r^2 = 45.84 \]

\[ r = 6.77 \]

\[ \tan \theta = \frac{6.73}{.74} \]

\[ \theta = \tan^{-1}\left(\frac{6.73}{.74}\right) \]

\[ \theta = 83.7 \]

direction\(=180-83.7=96.3\)

final answer: \(6.77/96.3^\circ\)
Example 30.2

The two forces (where lb = pound), 4/50° and -6/170° are applied to an object. Find the equilibrant of the two forces.

Note: to find the equilibrant of vectors (equilibrant means to cancel out)

1st: add the vectors
2nd: reverse the direction of the result
   a) if in polar, add 180°.
   b) if in rectangular, change the sign of both pieces.

4/50° + -6/170°

1) not rectangular

2) not 180 apart

3) skip
4) \[ 4/50^\circ \]

\[
\begin{align*}
\sin 50 &= \frac{y}{4} \\
y &= 4 \sin 50 \\
y &= 3.06 \\
x &= 4 \cos 50 \\
x &= 2.57
\end{align*}
\]

\((2.57, 3.06)\)

5) \((2.57+5.91, 3.06-1.04)\)

\(= (8.48, 2.02)\)

6) \[
\begin{align*}
r^2 &= 8.48^2 + 2.02^2 \\
r &= 8.72 \\
\theta &= \tan^{-1}\left(\frac{2.02}{8.48}\right) \\
\theta &= 13.4^\circ \\
\text{final sum: } 8.72/13.4^\circ \\
\text{equilibrant } &= (-8.48, -2.02) \text{ or } 8.72/193.4^\circ.
\]
Example 30.3

What vector force must be added to a force of 20/30° newtons to obtain a resultant of 25/0° newtons?

This means: \( \frac{20}{30°} + r/θ = \frac{25}{0°} \)

Break the vectors into rectangular form.

\[
\begin{align*}
\sin 30° &= \frac{y}{20} \\
y &= 20 \sin 30° \\
y &= 10
\end{align*}
\]

\[
\begin{align*}
\cos 30° &= \frac{x}{20} \\
x &= 20 \cos 30° \\
x &= 17.3
\end{align*}
\]

(17.3, 10)
Rewrite the problem in rectangular form.

\[(17.3,10) + (x,y) = (25,0)\]

So, \(17.3 + x = 25\) \hspace{1cm} And, \(10 + y = 0\)

\[x=7.7\]  \hspace{1cm}  \[y=-10\]

So, the unknown vector is \((7.7,-10)\) or convert to polar form.

\[r^2=7.7^2+10^2\]

\[r=12.6\]

\[\tan \theta = \frac{10}{7.7}\]

\[\theta = \tan^{-1}\left(\frac{10}{7.7}\right)\]

\[\theta = 52.4\]

direction: \(360-52.4=307.6\)

final answer: \(12.6/307.6^\circ\)
Overlapping Triangles

Redraw the triangles of interest separated.

Example 30.4

Givens: \( AB \cong DC \), \( BD \cong CA \)

Prove: \( \triangle ABD \cong \triangle DCA \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( AB \cong DC ) ( BD \cong CA ) ( AD \cong AD )</td>
<td>1) Given 2) Reflexive 3) SSS</td>
</tr>
<tr>
<td>3) ( \triangle ABD \cong \triangle DCA )</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Diagram of Overlapping Triangles} \]
Example 30.5

Givens: \( \overline{PQ} \cong \overline{RQ} \)
\( \overline{QY} \cong \overline{QX} \)

Prove: \( \triangle PQY \cong \triangle RQX \)

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<tr>
<td>1) ( \overline{PQ} \cong \overline{RQ} ) ( \overline{QY} \cong \overline{QX} )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle Q \cong \angle Q )</td>
<td>2) Reflexive</td>
</tr>
<tr>
<td>3) ( \triangle PQY \cong \triangle RQX )</td>
<td>3) SAS</td>
</tr>
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</table>
Example 30.6

Givens: \( \overline{XZ} \cong \overline{YZ} \)
\( \overline{XV} \perp \overline{YZ} \)
\( \overline{YU} \perp \overline{XZ} \)

Prove: \( \overline{XV} \cong \overline{YU} \)

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<td>1) ( \overline{XZ} \cong \overline{YZ} ) ( \overline{XV} \perp \overline{YZ} ) ( \overline{YU} \perp \overline{XZ} )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle ZVX ) &amp; ( \angle ZUY ) are right angles.</td>
<td>2) Def. of Perpendicular Lines</td>
</tr>
<tr>
<td>3) ( \angle ZVX \cong \angle ZUY )</td>
<td>3) Right angles are congruent</td>
</tr>
<tr>
<td>4) ( \angle Z \cong \angle Z )</td>
<td>4) Reflexive</td>
</tr>
<tr>
<td>5) ( \triangle ZVX \cong \triangle ZUY )</td>
<td>5) ASA</td>
</tr>
<tr>
<td>6) ( \overline{XV} \cong \overline{YU} )</td>
<td>6) CPCTC</td>
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Practice

a) Find the resultant of $7/\text{-}200^\circ$ and $5/276^\circ$. Write the answer in polar form.

$7/\text{-}200^\circ$.

$y = 7 \sin 20$

$y = 7 \sin 20$

$y = 2.39$

$x = 7 \cos 20$

$x = 7 \cos 20$

$x = 6.58$

(-6.58, 2.39)

$5/276^\circ$

$y = 5 \sin 84$

$y = 5 \sin 84$

$y = 4.79$

$x = 5 \cos 84$

$x = 5 \cos 84$

$x = 0.52$

(.52, -4.79)

$\sqrt{r^2} = 6.6^2 + 2.4^2$

$r = \sqrt{6.6^2 + 2.4^2}$

$r = 6.52$

$\theta = \tan^{-1} \left( \frac{2.4}{6.6} \right)$

$\theta = 21.6$

Direction: $180 + 21.6 = 201.6$

Final answer: $6.52/201.6^\circ$
b) 

**Givens:** \( \angle PQR \) & \( \angle SRQ \) are right angles 
\( PQ \cong SR \)

**Prove:** \( \triangle PQR \cong \triangle SRQ \)

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<td>1) ( \angle PQR ) &amp; ( \angle SRQ \Rightarrow \text{right}\angle's )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle Q \cong \angle R )</td>
<td>2) Right angles are congruent.</td>
</tr>
<tr>
<td>3) ( QR \cong QR )</td>
<td>3) Reflexive</td>
</tr>
<tr>
<td>4) ( \triangle PQY \cong \triangle RQX )</td>
<td>4) SAS</td>
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</table>

![Diagram of triangle PQR and triangle SRQ with right angles and congruent sides]
c) Solve: $\log_9 729 = c$

$9^c = 729$

Is 729 a power of 9?

Yes, $9^3$.

$9^c = 9^3$

$c = 3$
d) Solve:
\[
\begin{align*}
\begin{cases}
  x^2 - y^2 &= 8 \\
  2x^2 + y^2 &= 19
\end{cases}
\end{align*}
\]

\[
\begin{align*}
  3x^2 &= 27 \\
  x^2 &= 9 \\
  x &= \pm 3
\end{align*}
\]

For \( x = +3 \)

\[
\begin{align*}
  9 - y^2 &= 8 \\
  -y^2 &= -1 \\
  y &= \pm 1 \\
  (3,1) &\text{ & } (3,-1)
\end{align*}
\]

For \( x = -3 \)

\[
\begin{align*}
  9 - y^2 &= 8 \\
  -y^2 &= -1 \\
  y &= \pm 1 \\
  (-3,1) &\text{ & } (-3,-1)
\end{align*}
\]